

PARAMETRIC UNCERTAINTY IN A SIMPLE MODEL OF A SOCIAL-ECOLOGICAL NETWORK

Session "Archaeological Networks: Uncertainty, Missing Data, and Statistical Inference"

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Computer Applications in Archaeology 2017

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BACKGROUND

How can we better understand the two-way interaction between ancient cities (or towns, villages, camps ...) and their biophysical environments?

Lotka-Volterra style dynamical models represent the flow of energy between two populations, such as **predators and prey** in a trophic system or **cities and resources** in a social-ecological system.

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THE LOTKA-VOLTERRA MODEL

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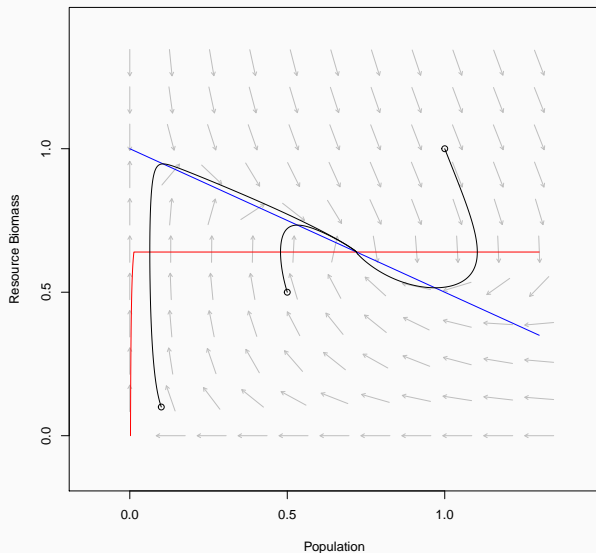
$$\begin{aligned}\dot{X} &= \overbrace{rX \left(1 - \frac{X}{K}\right)}^{\text{logistic growth}} - \overbrace{HXN}^{\text{harvest}} \\ \dot{N} &= \end{aligned} \tag{1}$$

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Under consumer-resource parametrization, the system will reach a **stable coexistence equilibrium** from any initial condition.





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Analysis

The effect of scaling and connection on the sustainability of a socio-economic resource system

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ORIGINAL ARTICLE

Living in a Network of Scaling Cities and Finite Resources

**Murad R. Qubbaj · Shade T. Shatters ·
Rachata Muneeppeerakul**

SCALING AND CONNECTIVITY

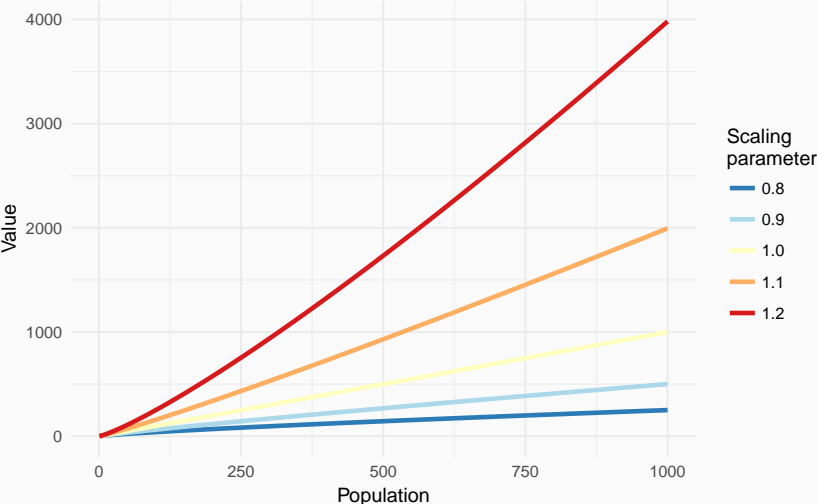


SCALING

POWER LAW SCALING PARAMETERS

Impact of power law scaling

Superlinear scaling in red, sublinear scaling in blue



$$\begin{aligned}\dot{X} &= rX \left(1 - \frac{X}{K}\right) - HXN \\ \dot{N} &= \frac{H}{E}XN - \frac{M}{E}N\end{aligned}\tag{2}$$

$$\begin{aligned}\dot{X} &= rX \left(1 - \frac{X}{K}\right) - HXN^\beta \\ \dot{N} &= \frac{H}{E} XN^\beta - \frac{M}{E} N\end{aligned}\tag{2}$$

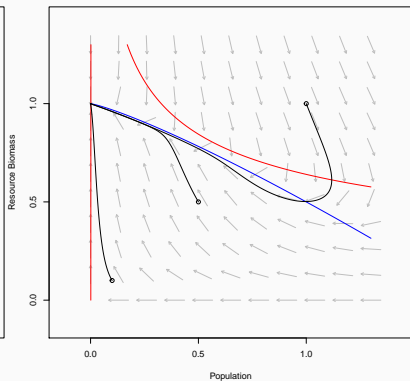
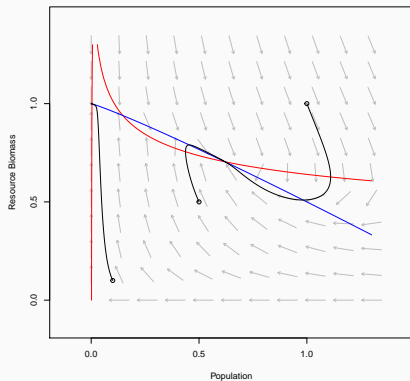
β Superlinear scaling of **harvest efficiency** with population size.

$$\begin{aligned}\dot{X} &= rX \left(1 - \frac{X}{K}\right) - HXN^\beta \\ \dot{N} &= \frac{H}{E} XN^\beta - \frac{M}{E} N^\alpha\end{aligned}\tag{2}$$

- β Superlinear scaling of **harvest efficiency** with population size.
- α Superlinear or sublinear scaling of **population maintenance requirement** with population size.

SCALING DYNAMICS

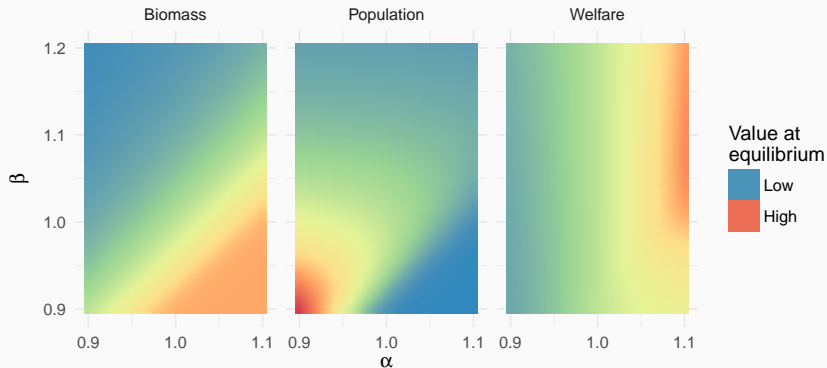
Including nonlinear scaling results in a **saddle-node bifurcation**. Weaker economies of scale introduce an **extinction equilibrium**, stronger economies of scale make extinction the only possible outcome.



SENSITIVITY TO SCALING PARAMETERS

Equilibrium sensitivity to power law scaling

All values normalized to $\alpha = \beta = 1$

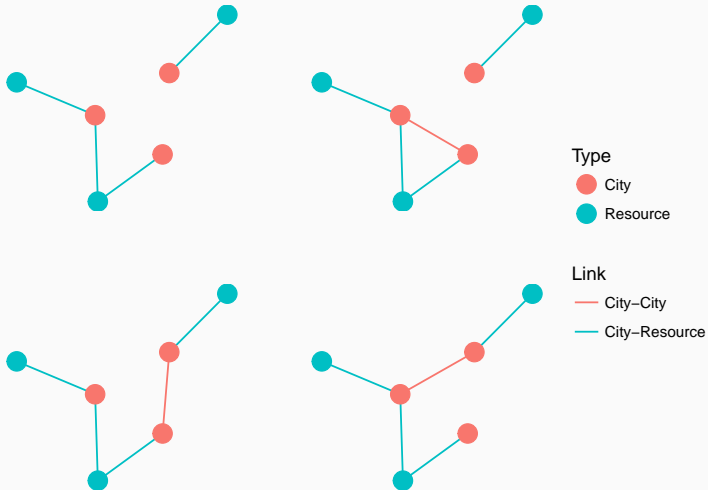


CONNECTIVITY

CONNECTIVITY STRUCTURE

Potential social–ecological connectivity structures

Under different parameterizations of ξ



Simulate city-resource and city-city connectivity by routing flows through adjacency matrices \mathbf{H} and ξ .

$$\dot{X}_i =$$

$$\dot{N}_j =$$

(3)

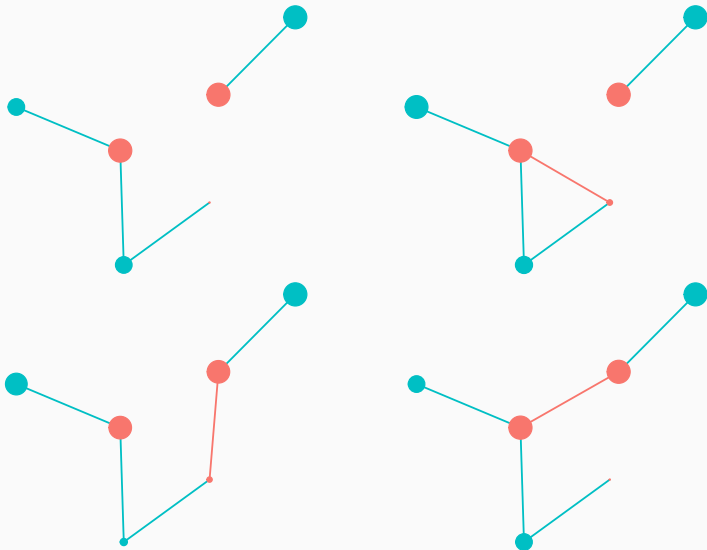
Simulate city-resource and city-city connectivity by routing flows through adjacency matrices \mathbf{H} and ξ .

$$\begin{aligned}
 \dot{X}_i &= rX_i \left(1 - \frac{X_i}{K}\right) - X_i \sum_j \overbrace{H_{ij} N_j^\beta}^{\text{resource flows to connected cities}} \\
 \dot{N}_j &= \frac{N_j^\beta}{E} \underbrace{\sum_i H_{ij} X_i}_{\text{flows from connected resource systems}} - \frac{M}{E} N_j^\alpha
 \end{aligned} \tag{3}$$

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 \dot{N}_j &= \frac{N_j^\beta}{E} \underbrace{\sum_i H_{ij} X_i}_{\text{flows from connected resource systems}} - \frac{M}{E} N_j^\alpha - \underbrace{\nu N_j}_{\text{migration out}} + \sum_k \underbrace{\xi_{jk} \nu N_k \frac{W_j}{\sum_l \xi_{lk} W_l}}_{\text{migration in}}
 \end{aligned} \tag{3}$$

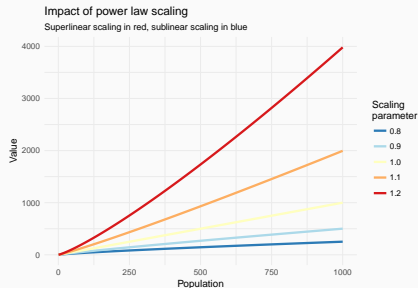
LONG-TERM IMPACTS OF CONNECTIVITY STRUCTURE



ARCHAEOLOGICAL IMPLICATIONS

We need **robust cross-cultural estimates** of scaling parameters

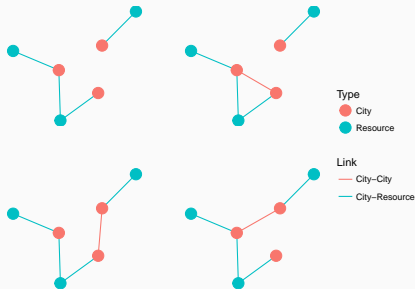
- We rarely know even if a given variable scales sublinearly or superlinearly with population size.



Social networks and transportation networks aren't sufficient for understanding dynamics, we need to think about **environmental flows** as well.

- Food webs, stream networks, precipitation teleconnections, etc., can't be ignored.

Potential social-ecological connectivity structures
Under different parameterizations of ξ



- Simple models of coupled **population and energy flows** can provide insights into prehistoric social networks.

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- **Nonlinear scaling** of socioeconomic factors with population size has a strong impact on the **sustainability** of ancient settlements.

- Simple models of coupled **population and energy flows** can provide insights into prehistoric social networks.
- **Nonlinear scaling** of socioeconomic factors with population size has a strong impact on the **sustainability** of ancient settlements.
- When scaling behaviors are present, even weak social or environmental **connectivity** can generate considerable **social-ecological complexity**.

QUESTIONS?