PARAMETRIC UNCERTAINTY IN A SIMPLE MODEL OF A SOCIAL-ECOLOGICAL NETWORK

Session "Archaeological Networks: Uncertainty, Missing Data, and Statistical Inference"

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BACKGROUND
How can we better understand the two-way interaction between ancient cities (or towns, villages, camps ...) and their biophysical environments?
Lotka-Volterra style dynamical models represent the flow of energy between two populations, such as predators and prey in a trophic system or cities and resources in a social-ecological system.

\[
\dot{X} = rX - \frac{HX}{K} - \frac{M}{X} \\
\dot{N} = H_N - \frac{M}{N}
\]  

(1)
Lotka-Volterra style dynamical models represent the flow of energy between two populations, such as predators and prey in a trophic system or cities and resources in a social-ecological system.

\[
\begin{align*}
\dot{X} &= \text{logistic growth} [rX \left(1 - \frac{X}{K}\right)] \quad \text{harvest} [HXN] \\
\dot{N} &= 
\end{align*}
\] (1)
Lotka-Volterra style dynamical models represent the flow of energy between two populations, such as predators and prey in a trophic system or cities and resources in a social-ecological system.

\[
\begin{align*}
\dot{X} &= rX \left(1 - \frac{X}{K}\right) - HXN \\
\dot{N} &= \frac{H}{E}XN - \frac{M}{E}N
\end{align*}
\]

(1)
Under consumer-resource parametrization, the system will reach a stable coexistence equilibrium from any initial condition.
Analysis

The effect of scaling and connection on the sustainability of a socio-economic resource system

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ORIGINAL ARTICLE

Living in a Network of Scaling Cities and Finite Resources

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SCALING
Impact of power law scaling
Superlinear scaling in red, sublinear scaling in blue

Scaling parameter
- 0.8
- 0.9
- 1.0
- 1.1
- 1.2
\[
\dot{X} = rX \left(1 - \frac{X}{K}\right) - HXN \\
\dot{N} = \frac{H}{E}XN - \frac{M}{E}N
\] (2)

Superlinear scaling of harvest efficiency with population size.
Superlinear or sublinear scaling of population maintenance requirement with population size.
MODELING SCALING

\[
\dot{X} = rX \left(1 - \frac{X}{K}\right) - HXN^\beta
\]

\[
\dot{N} = \frac{H}{E}XN^\beta - \frac{M}{E}N
\]

\( \beta \) Superlinear scaling of harvest efficiency with population size.
\[
\dot{X} = rX \left(1 - \frac{X}{K}\right) - HXN^\beta
\]

\[
\dot{N} = \frac{H}{E} XN^\beta - \frac{M}{E} N^\alpha
\]

\(\beta\) Superlinear scaling of harvest efficiency with population size.

\(\alpha\) Superlinear or sublinear scaling of population maintenance requirement with population size.
Including nonlinear scaling results in a **saddle-node bifurcation**. Weaker economies of scale introduce an **extinction equilibrium**, stronger economies of scale make extinction the only possible outcome.
Equilibrium sensitivity to power law scaling

All values normalized to $\alpha = \beta = 1$
CONNECTIVITY
Potential social–ecological connectivity structures

Under different parameterizations of $\xi$

CONNECTIVITY STRUCTURE

Type
- City
- Resource

Link
- City–City
- City–Resource
Simulate city-resource and city-city connectivity by routing flows through adjacency matrices $H$ and $\xi$.

$$\dot{X}_i =$$

$$\dot{N}_j =$$

(3)
Simulate city-resource and city-city connectivity by routing flows through adjacency matrices $H$ and $\xi$.

$$
\dot{X}_i = rX_i \left(1 - \frac{X_i}{K}\right) - X_i \sum_j H_{ij} N_j^\beta
$$

resource flows to connected cities

$$
\dot{N}_j = \frac{N_j^\beta}{E} \sum_i H_{ij} X_i - \frac{M}{E} N_j^\alpha
$$

flows from connected resource systems

(3)
Simulate city-resource and city-city connectivity by routing flows through adjacency matrices $H$ and $\xi$.

$$\dot{X}_i = rX_i \left(1 - \frac{X_i}{K}\right) - X_i \sum_j H_{ij} N_j^\beta$$

(resource flows to connected cities)

$$\dot{N}_j = \frac{N_j^\beta}{E} \sum_i H_{ij} X_i - \frac{M}{E} N_j^\alpha - \nu N_j + \sum_k \xi_{jk} \nu N_k \frac{W_j}{\sum_l \xi_{lk} W_l}$$

(flows from connected resource systems)

$$\text{migration out} \quad \text{migration in}$$

Equation (3)
ARCHAEOLOGICAL IMPLICATIONS
We need **robust cross-cultural estimates** of scaling parameters.

- We rarely know even if a given variable scales sublinearly or superlinearly with population size.
Social networks and transportation networks aren’t sufficient for understanding dynamics, we need to think about environmental flows as well.

- Food webs, stream networks, precipitation teleconnections, etc., can’t be ignored.

Potential social–ecological connectivity structures
Under different parameterizations of $\xi$
· Simple models of coupled population and energy flows can provide insights into prehistoric social networks.
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Nonlinear scaling of socioeconomic factors with population size has a strong impact on the sustainability of ancient settlements.
SUMMARY

- Simple models of coupled population and energy flows can provide insights into prehistoric social networks.
- Nonlinear scaling of socioeconomic factors with population size has a strong impact on the sustainability of ancient settlements.
- When scaling behaviors are present, even weak social or environmental connectivity can generate considerable social-ecological complexity.
Questions?